9 STRAIN MEASUREMENT

Direct measurement of mechanical stress is **not possible**. We can use Hooke's law, which defines relationship between mechanical stress and strain in elastic region of material behavior.

Hooke's law:

$$\sigma = E \cdot \varepsilon$$

Where:

σ *is mechanical stress [MPa]*
E *is modulus of elasticity [MPa]*
ε *is strain [-]*

Generally, devices such as **strain gauges** are used for strain measurement. The basic feature of strain gauges is the ability to measure very small changes in length, thus strain - typically 10^{-6} (1 μm/m).

Strain measurement is common part of many industries, in which principles of solid mechanics are used, for example: mechanical engineering, aerospace, medicine or civil engineering. In civil engineering the strain measurement is used for mechanical stress determination during experiments, numerical analysis verification or long-term monitoring of structures. The most used strain gauges types are:

- Extensometers (Mechanical strain gauges)
- Vibrating Wire strain gauges
- Resistive (Foil) strain gauges

9.1 Strain Gauges Used in This Course

9.1.1 Extensometer (Mechanical strain gauge)

If the material under strain gauge is being deformed, the deformation is transferred to the extensometer and causes change of its overall length. Change of extensometer length divided by its original (base) length results in strain:

$$\varepsilon_m = \frac{d}{L_0} \tag{9.1}$$

where:

ε_m *is strain determined with use of extensometer [μm/m],*
d *is actual length of extensometer (measurement on dial gauge) [μm],*
L_0 *original (base) length of strain gauge.*

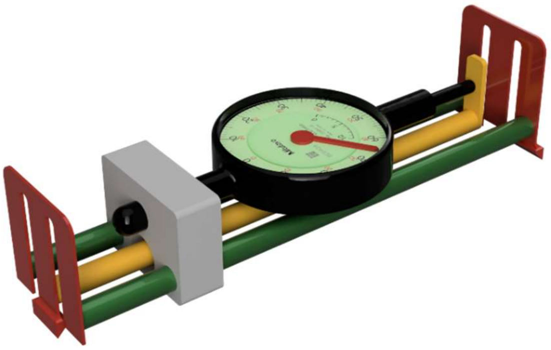


*Figure 9.1: Model of mechanical strain gauge (**Hollan's Bridge**) with dial gauge.*

9.1.2 Vibrating Wire strain gauge

This type of strain gauge consists of thin steel wire tensioned between two anchoring point attached to the deforming object and electro-magnetic coil. Deformation causes change in the length of the strain gauge hence the steel wire. Change of the wire length affect its natural vibration frequency, which is possible to determine the resulting strain from. Relationship between vibration frequency and strain is defined by:

$$\varepsilon_s = f^2 \cdot K \qquad 9.2$$

where:
ε_s *is strain measured by Vibrating Wire strain gauge [μm/m]*
f *is vibration frequency of wire [Hz]*
K *is constant of strain gauge.*

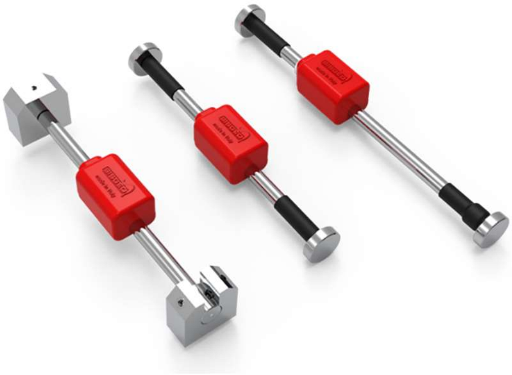


Figure 9.2: Types of Vibratin Wire strain gauges.

9.1.3 Resistive (Foil) strain gauge

Resistive (also Foil) strain gauges is the most common type of strain gauge used in experimental mechanics due its simple design, accuracy and low price. This type of strain gauge is made from resistive conducting grid between two layer of polymer foil. Strain gauge attached to the measured object (typically with the use of special glues) deforms with the material. Deformation of the measuring grid results in change of length and cross-sectional area of the grid which affects the electrical resistance of the strain gauge. From change in resistance and the original resistance of the strain gauge it is possible to calculate strain:

$$\frac{\Delta R}{R} = K \cdot \varepsilon \qquad 9.3$$

where:
R *is electrical resistance [Ω]*
K *is strain gauge constant [-]*
ε *is mechanical strain [μm/m].*

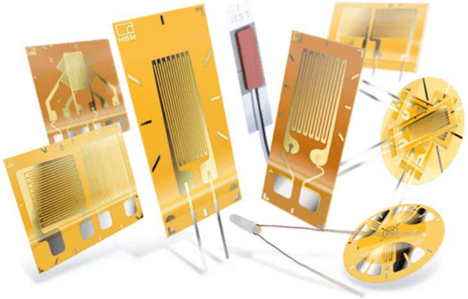


Figure 9.3: Example of HBM strain gauges catalog

In practical measurement of strain with the use of resistive strain gauges measuring and calculations are provided by measuring amplifier which returns values of strain directly (typically in $\mu m/m$ units).

Wheatstone Bridge

In practical application of resistive strain gauges direct resistance change measuring is not possible. The main reason is the fact, that changes in resistance are very low and direct measurement cannot be accurate. For this reason the Wheatstone bridge is commonly used. Wheatstone bridge consists of passive resistors and active strain gauges arranged to the circuit. When bridge is excited by voltage, very small changes in resistance of bridge elements results in measurable voltage output between bridge arms. Based on number of active strain gauges (SG), four basic types of Wheatstone bridges are used (Fig. 9.4):

- Full bridge (4 active SGs)
- Quarter bridge (1 active SG, 3 complementary resistors)
- Half bridge (2 active SGs, 2 complementary resistors)
    - Diagonal (active SGs on opposite bridge arm)
    - Parallel (active SGs on same bridge arm)

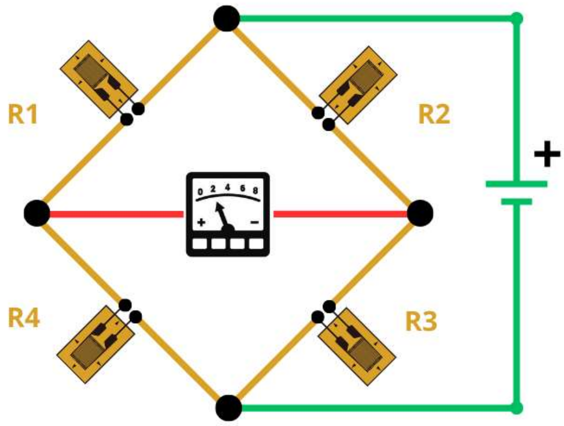
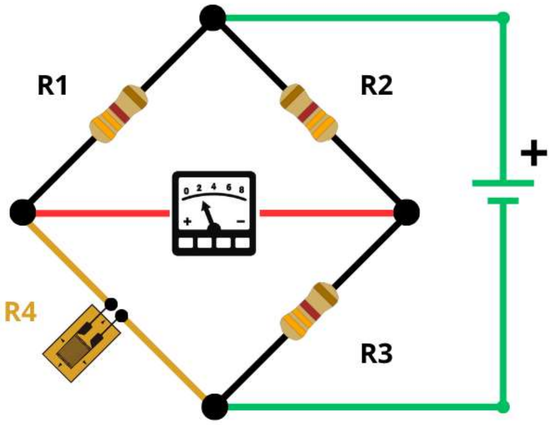
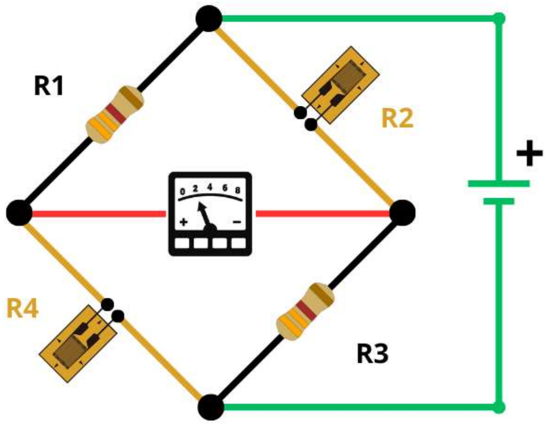


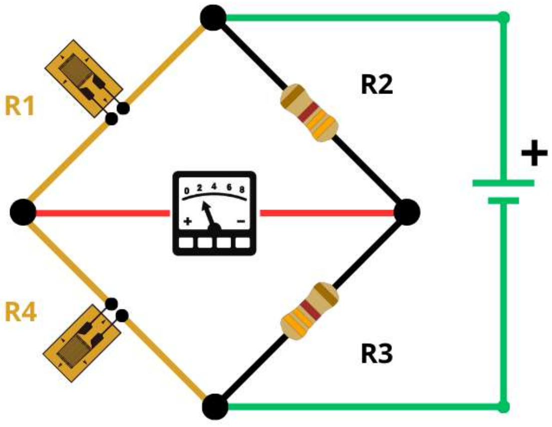
Figure 9.4a: Full bridge



Obr.9.4b: Quarter bridge



Obr.9.4c: Half bridge - diagonal



Obr.9.4d: Half bridge - parallel

Change in resistance (strain) of active strain gauges affects the overall measured strain. Specific type off bridge arrangement, orientation of the SGs on the measured object and loading conditions yield in specific overall strain measurement. General equation for resulting strain is:

$$\varepsilon = +\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 \tag{9.4}$$

where*:*

ε *is overall strain [μm/m]*
ε_1 *is strain measured by SG1 [μm/m]*
ε_2 *is strain measured by SG2 [μm/m]*
ε_3 *is strain measured by SG3 [μm/m]*
ε_4 *is strain measured by SG4 [μm/m].*

9.2 Task I: Tensile Loading – Strain Gauges Types Comparison

This experiment is designed to demonstrate basic types of strain gauges usage in practical measurement. Experiment will be executed by direct tensile loading test of a GFRP (Glass Fibre Reinforced Polymer) rod loaded with steel ballast.

Rod properties:

GFRP rod – PREFA KOMPOZITY, a.s.

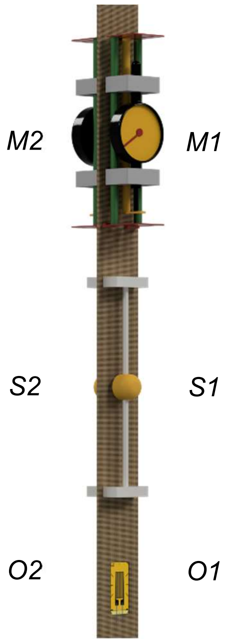
cross-section *b x t*: **50,70 x 3,30 mm**

modulus of elasticity *E*: **24 000 MPa**

Ballast properties:

weight of ballast bracket: 13,50 kg
weight of single ballast rod: 10,40 kg

overall weight: 138,30 kg (1 bracket + 12 rods)



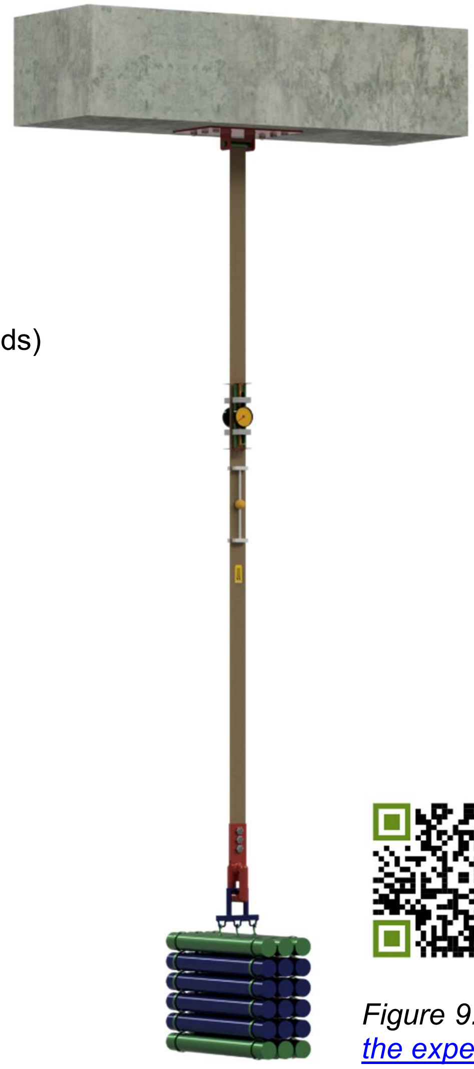


Figure 9.5: Model of the experiment

In this experiment (elongation) strain of the rod will be measured by:

- Extensometers (Hollan's bridges with base length of L_0 **= 120 mm**) with dial gauges *Mitutoyo* with accuracy of 0,001 mm
- Vibrating Wire strain gauges *Gage Technique International Ltd.* typu *TSR/5.5* with specific constant of SGs of $K = 3{,}025 \cdot 10^{-3}\ \mu m/m \cdot Hz^2$
- Resistive (Foil) SGs *HBM 1-LY11-10/350*

<u>SGs labelling:</u>

- Extensometers – *M1, M2*
- Vibrating Wire SGs – *S1, S2*
- Resistive SGs – *O1, O2*

Aim of this task is to determine theoretical stress and then verify theoretical calculation with experimental measurement and compare results.

9.2.1 Measurement procedure

- Values reading before loading (initial reading) – $d_0, f_0, \varepsilon_{o,0}$
- Applying of the ballast
- Loaded rod values reading - d, f, ε_o
- Unloading of the rod
- Values reading after unloading – $d_0', f_0', \varepsilon_{o,0}'$

9.2.2 Theoretical stress calculation

In case of the uniaxial tensile loading, the mechanical stress is determined by:

$$\sigma_{N,theor} = \frac{N}{A} \qquad 9.5$$

where:

$\sigma_{N,theor}$ *theoretical stress [MPa]*
N *normal force [N]*
A *cross-sectional area of the rod [mm^2].*

In case of normal force calculation gravitational acceleration of g **= 9,81 m/s²** is used.

9.2.3 Determination of the stress based on experimental measurement

All calculations of the experimental determined stress values will be based on Hooke's law formula, measured strain and modulus of elasticity of GFRP rod. The strain used in the calculations must represent loading procedure during experiment. The resulting strain will be determined as a difference between loaded state and state before loading (initial reading).

Extensometers
Strain from extensometers $\boldsymbol{\varepsilon}_m$ will be calculated according to *9.1.*
Vibrating Wire SGs
Strain from this type of SG $\boldsymbol{\varepsilon}_s$ will be calculated according to *9.2.*
Resistive SGs
Strain from this type of SG $\boldsymbol{\varepsilon}_o$ will be directly determined from measuring amplifier.

Resulting strain for stress calculations will be calculated as mean value for the particular strain gauge type.

9.3 Task II: Bending – Configuration of the Strain Gauges

Based on specific configuration of the resistive strain gauges in the Wheatstone bridge circuit it is possible to achieve for example: temperature effect compensation, increase sensitivity of the measuring chain, or separate types of the load (e.g. normal force from bending moment). On the other hand, incorrectly chosen configuration can negatively affect measured results.

In this task three configuration of the resistive SGs and Wheatstone bridge configuration will be presented. Measuring will be executed on cantilever beam made from the steel hollow structural section.

Cantilever properties:

Cantilever beam from steel hollow structural section:

width of section b: 50 mm

height of section h: 30 mm

wall thickness t_w: 3,0 mm

length of beam l: 1 500 mm

distance from support to measuring point a: 20 mm

moment of inertia I_z: **$5{,}70 \cdot 10^4$ mm^4**

section modulus W_y: **3 800 mm^3**

modulus of elasticity of steel: **210 000 MPa**

Ballast properties:

weight of ballast bracket: 11,72 kg
weight of single ballast rod: 10,40 kg

overall weight: 35,52 kg (1 bracket + 2 rods)

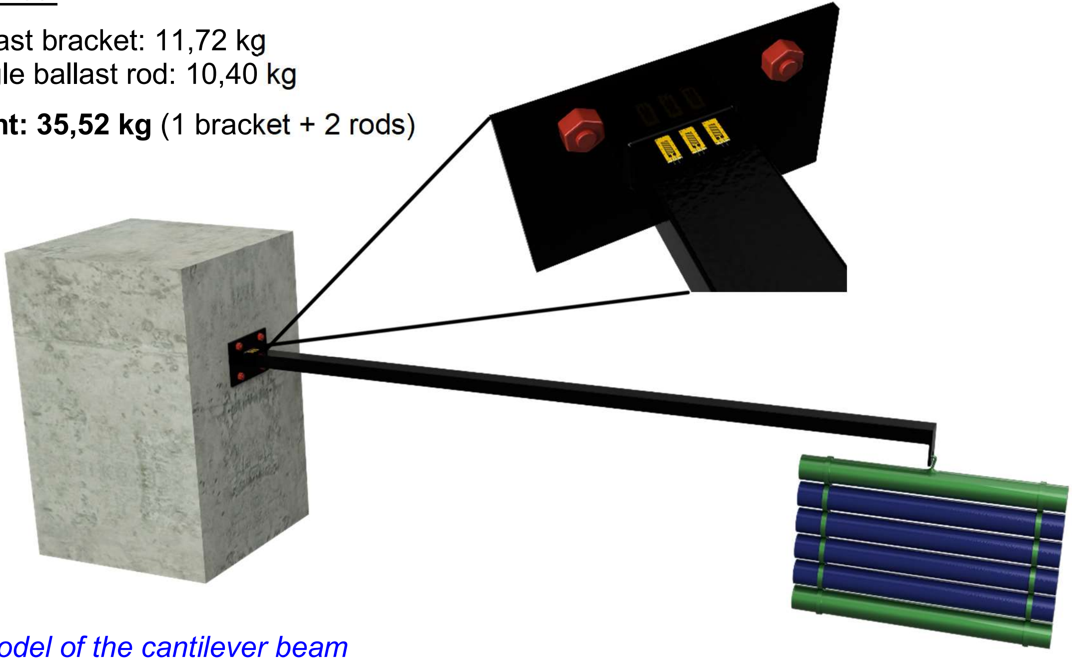


Figure.9.6: Model of the cantilever beam

Loading of the cantilever beam on the cantilever end will cause bending moment ***M***, thus mechanical stress $\boldsymbol{\sigma_x}$. The maximal value of the bending moment and stress will be at the support of the beam. Based on the direction of loading, positive (tensile) stress occurs in the upper part of the beam, on the other hand bottom part of the beam will be exposed to the negative (compressive) stress. But due to axial symmetry of the cross-section, both of the stress values will be equal.

For strain measurement 6 resistive strain gauges (type *HBM 1-LY11-6/350)* will be used in total. Three SGs will be attached on upper surface and three on bottom surface. Resulting strain will be measured according to the formula ***9.4***. Configuration of the SGs in the Wheatstone bridge circuit will be following:

- **Quarter bridge** – top and bottom SG connected in the separate bridge circuit. Measured strain will be read directly and separately for both of the SGs – $\boldsymbol{\varepsilon_{1/4h}}$ (SG on the upper surface), $\boldsymbol{\varepsilon_{1/4d}}$ (SG on the bottom surface).

- **Half bridge – parallel** – upper **R1** and bottom **R4** SG connected together in to parallel half bridge. In theory, resulting strain $\boldsymbol{\varepsilon_{1/2\_par}}$ will be twice as high as real strain acting on the structure, according to:

$$\varepsilon_{1/2\_par} = +\,\varepsilon_1 - (-\varepsilon_4)$$

- **Half bridge – diagonal** - both of the SGs (**R2** and **R4**) arranged to the diagonal half bridge circuit. Resulting strain $\boldsymbol{\varepsilon_{1/2\_diag}}$ according to the formula 9.4 will be theoretically zero:

$$\varepsilon_{1/2\_diag} = -\,\varepsilon_2 - (-\varepsilon_4)$$

9.3.1 Measurement procedure

Strain reading from each type of bridge configuration will be carried out directly from the measuring laptop and data acquisition software. Initial reading on the start of the experiment will be zero. Then will be determined strain on the fully loaded beam. Also, check measurement after unloading will be performed.

Steps of experiment:

- Start of the measurement
- Applying of the ballast
- Strain value reading – ($\varepsilon_{1/4h}$, $\varepsilon_{1/4d}$, $\varepsilon_{1/2\_par}$, $\varepsilon_{1/2\_diag}$)
- Unloading
- Strain value reading after unloading– ($\varepsilon_{1/4h}$, $\varepsilon_{1/4d}$, $\varepsilon_{1/2\_par}$, $\varepsilon_{1/2\_diag}$)

9.3.2 Theoretical stress calculation

In case of bending, theoretical stress can be determined by:

$$\sigma_{M,teor} = \frac{M}{W} \qquad 9.6$$

where:

$\sigma_{N,teor}$ *theoretical stress from bending [MPa]*
M *bending moment [Nm] $\cdot 10^3$*
W *section modulus [mm^3].*

In case of force calculation gravitational acceleration of **g = 9,81 m/s^2** is used.

9.3.3 Theoretical strain calculation

For preliminary theoretical strain calculation, Hooke's law will be used.

Report **STRAIN MEASUREMENT**	**T**
Instructor:	

Task I: Tensile Loading – Strain Gauges Types Comparison

Task I – Measured values

	extensometers		Vibrating Wire SGs		Resistive SGs	
	M1	M2	S1	S2	O1	O2
	d_1	d_2	f_1	f_2	ε_1	ε_2
	[mm]	[mm]	[Hz]	[Hz]	[μm/m]	[μm/m]
initial reading						
loaded structure						
unloaded structure						

Task I – Calculation of experimental based strain and stress

	extensometers		Vibrating Wire SGs		Resistive SGs	
	M1	M2	S1	S2	O1	O2
	$\varepsilon_{m,1}$	$\varepsilon_{m,2}$	$\varepsilon_{s,1}$	$\varepsilon_{s,2}$	$\varepsilon_{o,1}$	$\varepsilon_{o,2}$
	[μm/m]	[μm/m]	[μm/m]	[μm/m]	[μm/m]	[μm/m]
initial reading						
loaded structure						
difference						
mean ε [μm/m]						
stress σ [MPa]						

Task I – Theoretical stress calculation $\sigma_{N,teor}$

Task I – Comparison of the results

		theoretical stress	extensometers	Vibrating Wire SGs	Resistive SGs
stress ***σ***	[MPa]				
difference	[%]	0			

Conclusion:

Task II: Bending – Configuration of the Strain Gauges

Task II – Measured values

	quarter bridge		half bridge - parallel	half bridge - diagonal
	upper	bottom		
	$\varepsilon_{1/4h}$	$\varepsilon_{1/4d}$	$\varepsilon_{1/2\_par}$	$\varepsilon_{1/2\_diag}$
	[μm/m]	[μm/m]	[μm/m]	[μm/m]
initial reading	0	0	0	0
loaded structure				
	mean:			
unloaded structure				
	mean:			

Task II – Theoretical stress calculation $\sigma_{M,teor}$

Task II – Theoretical strain calculation ε_{teor}

Task II – Comparison of the results

		theoretical stress	extensometers	Vibrating Wire SGs	Resistive SGs
stress $\boldsymbol{\sigma_x}$	[MPa]				
difference	[%]	0			

Conclusion: