| MODULUS OF ELASTICITY OF CONCRETE | E |
|---|---|

Within the limits of Hooke's law, the modulus of elasticity is the proportionality constant between stress and strain. For concrete, it corresponds to the slope of the straight line fitted to the initial portion of the stress-strain diagram in compression. In general, the modulus of elasticity is a specific measure of the stiffness of a solid material in compression (or tension).

This is a relatively important property of concrete that affects the behavior of structures, particularly slender or prestressed ones (with respect to deflections, creep, etc.).

# 1 Determination of the Static Modulus of Elasticity of Concrete in Compression

The static modulus of elasticity is determined from the deformations that occur under known loading conditions.

## 1.1 Measurement Principle

A test specimen for determining the static modulus of elasticity of concrete in compression, according to ISO 1920-10, can take the shape of a prism or a cylinder. Its slenderness, i.e., the ratio of length to cross-sectional dimension, must be at least 2.

The testing principle involves loading the test specimen in a compression testing machine while simultaneously measuring the resulting deformations. The test is performed cyclically—at least three cycles are carried out, during which the specimen is subjected to varying loads. The load levels are chosen so as not to completely unload the specimen and to ensure that loading remains within the elastic range (within the validity range of Hooke's law). According to ISO 1920-10, the basic (lower) stress level is always $\sigma_b = 0.5\,\text{MPa}$ ($\text{N/mm}^2$), and the upper stress level should correspond to one-third of the compressive strength of the tested concrete, $\sigma_a = f_c/3\,\text{MPa}$ ($\text{N/mm}^2$).

## 1.2 Determination of Compressive Strength of Comparative Specimens and Load Levels

Before testing, the compressive strength of the tested concrete must be known to correctly set the upper load stress level $\sigma_a$. According to ISO 1920-10, the compressive strength of concrete is determined using three comparative specimens of the same size and shape, which were manufactured and cured in the same manner as the specimens used for determining the static modulus of elasticity.

From the average compressive strength $f_c$ of the comparative specimens, the stress level used for determining the static modulus of elasticity ($f_c/3$) is calculated. During testing, where the stress level in the tested concrete is cyclically changed from the basic stress $\sigma_b$

to the upper stress $\sigma_a$, the deformation response of the specimen is monitored. Based on the stress levels already determined and the loaded area $A$ of the test specimen, the basic force $F_b$ and the upper force $F_a$ are calculated:

$$F_b = 0.5 \cdot A \tag{1}$$

$$F_a = \frac{f_c}{3} \cdot A \tag{2}$$

where $F_b$ is the basic force in N, 0.5 is the basic stress in MPa, $A$ is the loaded area of the test specimen (calculated from the determined dimensions of the specimen) in mm$^2$, $F_a$ is the upper force in N, and $f_c$ is the average compressive strength of the comparative specimens in MPa.

The force values must be adjusted by rounding according to the scale of the testing machine used—in this case, to whole kN. The actual values of the basic stress $\sigma_b$ and the upper stress $\sigma_a$ must be recalculated based on the adjusted force values.

## 1.3 Preparation of Test Specimens and Centering

The test specimen must be prepared before the test itself. Instruments for measuring length changes (in this exercise, mechanical extensometers with digital indicators) must be attached to the sides of the specimen so that the measured points are equidistant from both ends of the specimen and lie along its longitudinal axis. Deformations (or strains) must be measured on at least two opposite sides of the specimen, see Figure 1.

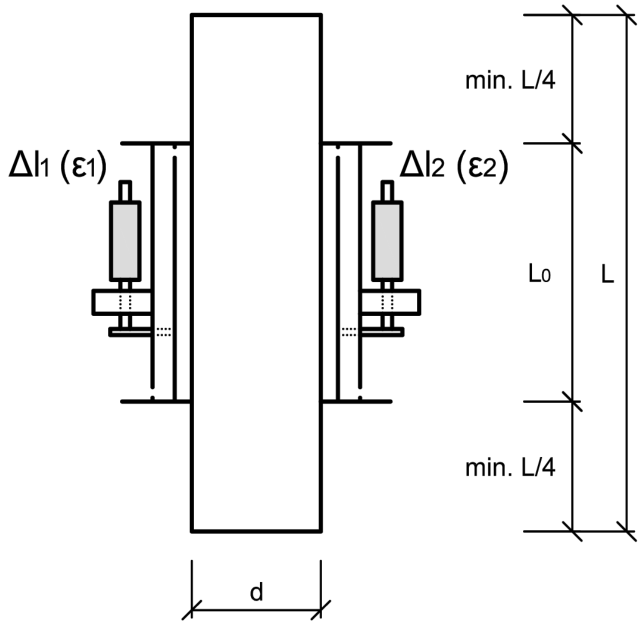


Figure 1: Placement of Mechanical Extensometers on the Test Prism

When loading the test specimen, it is important to ensure that the load is applied as evenly as possible across the entire loaded area, i.e., that the loading is centric. During loading, the centricity of the specimen must be verified. The time progression of centering is graphically depicted in Figure 2.

The test specimen, with extensometers attached, is placed in the testing machine (Point 1A). Next, the specimen is loaded to the basic stress level $\sigma_b$, and after 60 seconds, readings are taken from all devices (Point 1B). The stress is then gradually increased to the upper stress level $\sigma_a$, and after another 60 seconds, readings are taken again (Point 1C). If the individual strains $\varepsilon$ (or deformations $\Delta l$) differ by more than 20% from their average value, the test specimen is not centered and must be fully unloaded.

Based on the readings from the extensometers, the position of the test specimen in the testing machine is adjusted, and the entire procedure is repeated (Points 2A to 2C). If it is determined that the specimen is still not centered, the process is repeated again. Once centering is successful, the test specimen is unloaded to the basic stress level $\sigma_b$, and the entire test is completed.

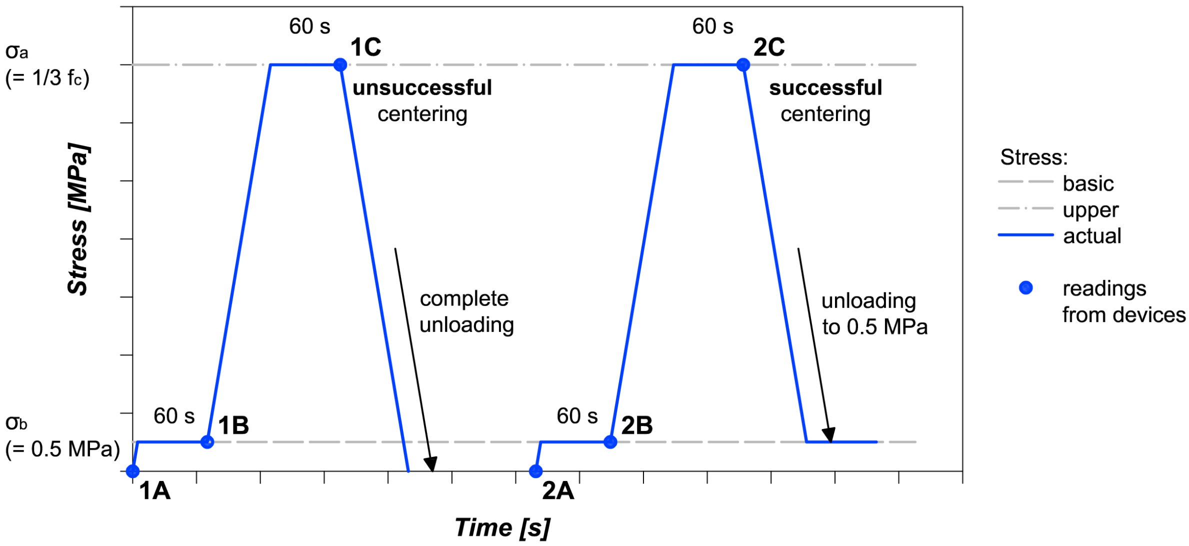


Figure 2: Graphical Representation of Specimen Centering

## 1.4 Loading the Test Specimen

The time progression of the static modulus of elasticity test is graphically shown in Figure 3. After successfully centering the specimen, the load is maintained at the basic stress level $\sigma_b$ for 60 seconds. After recording the extensometer readings, the load is gradually increased to the upper stress level $\sigma_a$, where readings are again taken after 60 seconds.

One cycle consists of applying the lower stress level for 60 seconds, recording the readings, applying the upper stress level for 60 seconds, and recording the readings. The entire test comprises at least two preliminary loading cycles, followed by a measurement loading cycle. The values recorded during the final cycle are used to calculate the static modulus of elasticity $E_c$. The cycle during which centering was successfully verified can be considered the first preliminary cycle.

After the measurements are completed, the load on the test specimen is increased at the prescribed rate until failure. If the compressive strength of the test specimen $\sigma_c$ differs from the strength of the comparative specimens $f_c$ by more than 20%, this must be noted in the report.

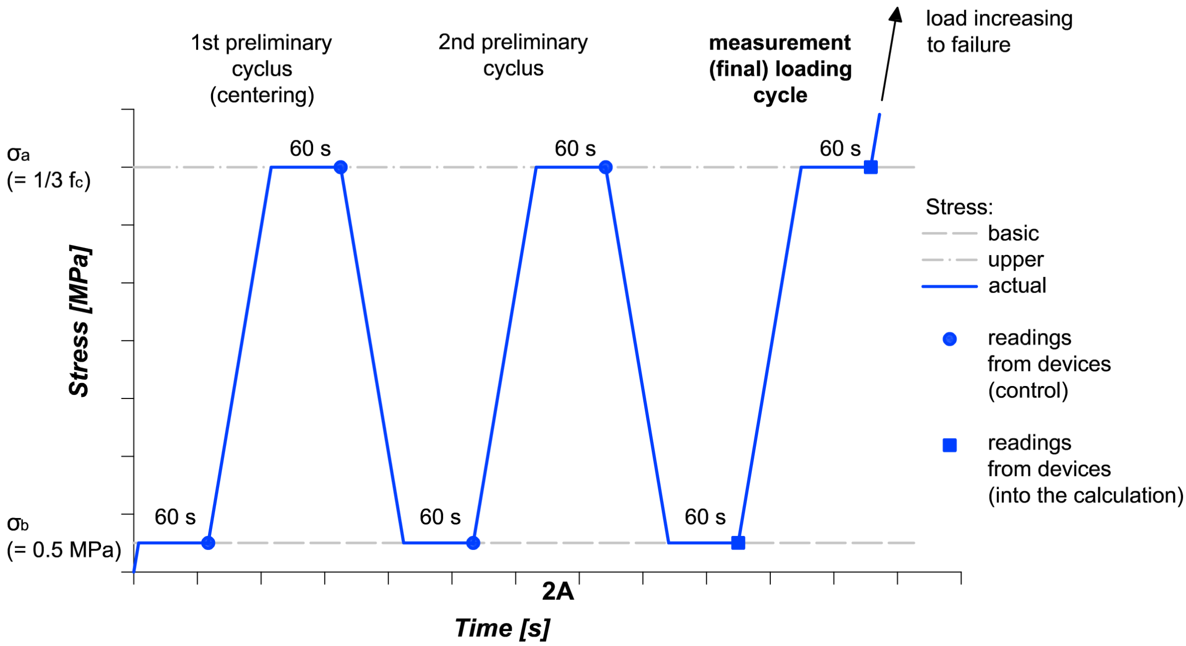


Figure 3: Graphical Representation of the Loading Progression of the Test Specimen

## 1.5 Processing the Measured Data, Calculation of Stress, and Strains

The length changes of the individual bases (Extensometers 1 and 2) $\Delta l_1$ and $\Delta l_2$ are given by the changes in the readings of the displacement gauges:

$$\Delta l_{1(2)} = l_a - l_b \tag{3}$$

where $\Delta l_{1(2)}$ is the length change of the base of the 1st (2nd) extensometer in mm (or $\mu$m), $l_a$ is the displacement gauge reading at the upper stress level in mm (or $\mu$m), and $l_b$ is the displacement gauge reading at the basic stress level in mm (or $\mu$m).

The average length change determined by the extensometers $\Delta l$ is calculated using the following formula:

$$\Delta l = \frac{\Delta l_1 + \Delta l_2}{2} \tag{4}$$

Care must be taken in further calculations to ensure that the length change $\Delta l$ of the test specimen is expressed in the correct units (mm or $\mu$m). The average strain $\Delta\varepsilon$ is calculated as follows:

$$\Delta\varepsilon = \frac{\Delta l}{L_0} \tag{5}$$

where $\Delta l$ is the average length change of the bases in mm or $\mu$m (see Equation 4) and $L_0$ is the gauge length (bases lenght) of the of the mechanical extensometer (in this exercise, extensometers with $L_0 = 200$ mm will be used).

### 1.6 Calculation of Modulus of Elasticity

The modulus of elasticity is defined as the ratio of the stress change to the corresponding strain change, in accordance with Hooke's law. The average strain change is calculated from the measurements during the loading cycle, as per Equations (4) and (5). The stress change is determined as the difference between the basic and upper stress levels:

$$\Delta\sigma = \sigma_a - \sigma_b \tag{6}$$

The static modulus of elasticity in compression $E_c$ is then calculated as:

$$E_c = \frac{\Delta\sigma}{\Delta\varepsilon} \tag{7}$$

where $\Delta\sigma$ is the stress difference during loading in MPa, and $\Delta\varepsilon$ is the average strain change between the upper and basic stress levels in **m/m**. The resulting modulus of elasticity $E_c$ is expressed in **GPa**, rounded to the nearest 0.1 GPa.

## 2 Evaluation

The dynamic modulus of elasticity differs from the static modulus of elasticity. The ratio between the static and dynamic values (reduction coefficient) primarily depends on the quality and age of the concrete. Generally, the lower the quality (or younger) the concrete, the lower the ratio between the static and dynamic modulus of elasticity (it can be less than 0.6). Conversely, high-quality (and sufficiently matured) concretes can have this ratio exceed 0.9.

Additionally, the two dynamic methods (ultrasonic pulse velocity vs. resonance – see laboratory reports n. 8 and 9) do not yield identical results. Differences in results from these standardized procedures are also noted in ČSN 73 2011, which provides different reduction coefficients $\kappa_u$ and $\kappa_r$ for the same strength classes of concrete, see Table 1.

During the exercise, the reduction coefficient $\kappa_u$ for the tested concretes will be determined using the following formula:

$$\kappa_u = \frac{E_c}{E_c u} \tag{8}$$

Table 1: Reduction coefficients $\kappa_u$ and $\kappa_r$ for converting the dynamic modulus of elasticity to the static modulus according to ISO 73 2011.

| **Concrete Class** | $\kappa_u$ | $\kappa_r$ |
|---|---|---|
| C 8/10 | 0.62 | 0.81 |
| C 12/15 | 0.71 | 0.86 |
| C 16/20 | 0.76 | 0.88 |
| C 25/30 | 0.81 | 0.90 |
| C 30/37 | 0.83 | 0.91 |
| C 35/45 | 0.86 | 0.93 |
| C 40/50 | 0.88 | 0.94 |
| C 45/55 | 0.90 | 0.95 |

where $\kappa_u$ is the reduction coefficient, $E_{cu}$ is the dynamic modulus of elasticity determined using the ultrasonic pulse velocity method (see laboratory measurement record 8), and $E_c$ is the static modulus of elasticity.

The result of the static modulus of elasticity test can be compared with the reference (i.e., average) modulus values provided for each strength class in EN 1992-1-1 ed. 2 (Eurocode 2), see Table 2.

Table 2: Reference values of modulus of elasticity according to Eurocode 2.

| **Concrete Strength Class** | $E_cm$ **[GPa]** |
|---|---|
| C 12/15 | 26.0 |
| C 16/20 | 27.5 |
| C 20/25 | 29.0 |
| C 25/30 | 30.5 |
| C 30/37 | 32.0 |
| C 35/45 | 33.5 |
| C 40/50 | 35.0 |
| C 45/55 | 36.0 |
| C 50/60 | 37.0 |

# MODULUS OF ELASTICITY OF CONCRETE | E

**Instructor:**

Determination of the static modulus of elasticity $E_c$ of tested concrete. The necessary information for the determination of $E_c$ will be provided by the instructor.

Calculation of stress levels $\sigma_b$ and $\sigma_a$ and corresponding forces $F_b$ and $F_a$:

| Comparative prism | a [mm] | b [mm] | $F_c$ [kN] | $f_c$ [MPa] |
|---|---|---|---|---|
| LWC2 | | | | |
| OC2 | | | | |

| Prism | a [mm] | b [mm] | $\sigma_b$ [MPa] | $1/3 \cdot f_c = \sigma_a$ [MPa] | $F_b$ [kN] | $F_a$ [kN] |
|---|---|---|---|---|---|---|
| LWC1 | | | | | | |
| OC1 | | | | | | |

Measurement of deformations:

| | $\Delta l_1$ | $\Delta l_2$ | [ ] |
|---|---|---|---|
| 1. p. c. (cen.) | | | ($F_B$) |
| | | | ($F_A$) |
| 2. p. c. | | | ($F_B$) |
| | | | ($F_A$) |
| m. c. | | | ($F_B$) |
| | | | ($F_A$) |

| | $\Delta l_1$ | $\Delta l_2$ | [ ] |
|---|---|---|---|
| 1. p. c. (cen.) | | | ($F_B$) |
| | | | ($F_A$) |
| 2. p. c. | | | ($F_B$) |
| | | | ($F_A$) |
| m. c. | | | ($F_B$) |
| | | | ($F_A$) |

Calculation of the static modulus of elasticity $E_c$:

Calculation of compressive strength of prisms after the $E_c$ test:

(Determine whether the test meets the standard requirements by checking if the actual compressive strength does not differ from the expected compressive strength by more than 20 %.)

| | | |
|---|---|---|
| **Concrete:** | | |
| **$F_{max}$ [kN]:** | | |
| **$f_c$ – actual (on tested prisms) [MPa]:** | | |
| **$f_c$ – expected (on comparative prisms) [MPa]:** | | |
| **Difference [%]:** | | |

Calculate the reduction coefficient $\kappa_u$ for the tested concrete and compare it with the normative values (see Table 1). (The values of $E_{cu}$ can be found in the laboratory report no. 8.)

| | | |
|---|---|---|
| Concrete: | | |
| $E_c$ [GPa]: | | |
| $E_{cu}$ [GPa] (Laboratory report no. 8): | | |
| $\kappa_u$ [-]: | | |
| Strength class of concrete: | | |
| $\kappa_{u,\text{standard}}$ [-]: | | |
| Difference: | | |

Compare the calculated static moduli of elasticity with the values from the EN 1992-1-1 standard (see Table 2).

| | | |
|---|---|---|
| Concrete: | | |
| $E_c$ [GPa]: | | |
| $E_{cm}$ [GPa] – Eurocode 2: | | |
| Difference: | | |

Conclusion:

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Tests conducted and report prepared by:

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